Sequences of bifurcations and transition to Chaos in an Optical-Processing Element

Ana González-Marcos & J.A. Martín-Pereda

E.T.S.I. Telecomunicación. Ciudad Universitaria, 28040 - Madrid, Spain

Abstract. Digital chaotic behaviour in an Optical-Processing Element is reported. It is obtained as the result of processing two fixed trains of bits. Period doublings in a Feigenbaum-like scenario have been obtained. A new method to characterize digital chaos is reported

1. Introduction

An optically-programmable digital circuit has been already reported by us, [1]-[2], as a Programmable Logic Gate. A brief description on its method of operation, as well as the way it has been implemented, can be found there.

As is very well known from the literature, there are several situations where a chaotic behaviour arises from electrical and electronic circuits. Most of the results concern, and are related, to analogue signals. Their characteristics have been studied by conventional methods employed in any other nonlinear phenomena.

A very different situation is present when the circuit operates with digital signals and the possible chaotic result is a signal composed of "zeroes" and "ones". Hence, the main objective of this paper is to present a new method to obtain the above mentioned type of chaotic signals as well as an alternative way to their study.

2. General structure of the Optical-Processing Element with feedback.

The general scheme of the Cell has been previously reported in several places [1]-[3]. A logic behaviour was reported showing the possibility to obtain up to fourteen pairs of logic functions from two digital inputs. Two control gates allow the change from one type of logic output to another. The cell was implemented with optical components although the non-linear devices **P** and **Q**, namely an "on-off" and a "SEED-like", were simulated with optoelectronic methods (see Fig. 1). The output of each one of them corresponds to the two final outputs, O_1 and O_2, of the cell. The possible inputs to the circuit are four. Two of them are for input data, $\mathbf{I}_1$ and $\mathbf{I}_2$, and the other two, g and h, for control signals. The corresponding inputs to the non-linear devices, **P** and **Q**, are based on these signals plus some others coming from inside the cell.

The practical implementation we have carried out of the processing element has been based on an optoelectronic configuration. Lines in Fig. 1 represent optical multimode fibers. The indicated blocks, placed in order to combine the corresponding signals, are conventional optical couplers. In this way, the inputs arriving to the above mentioned **P** and **Q** devices, are multilevel signals. These devices have been simulated electronically. Optical signals were converted to electrical by conventional photodiodes and, after processing, converted again to optical signals by LEDs. More details can be seen in reference [3].

A new situation appears when some type of feedback is added to the cell. Moreover, in order to have the possibility to work with some more parameters, a time delay has been added to the feedback. Another time delay has been introduced inside the own cell. This time corresponds to response time of the non-linear devices that, in our previous case, were optoelectronic simulations. The general configuration appears in Fig. 1, where these delays, as well as the whole cell configuration, is shown.

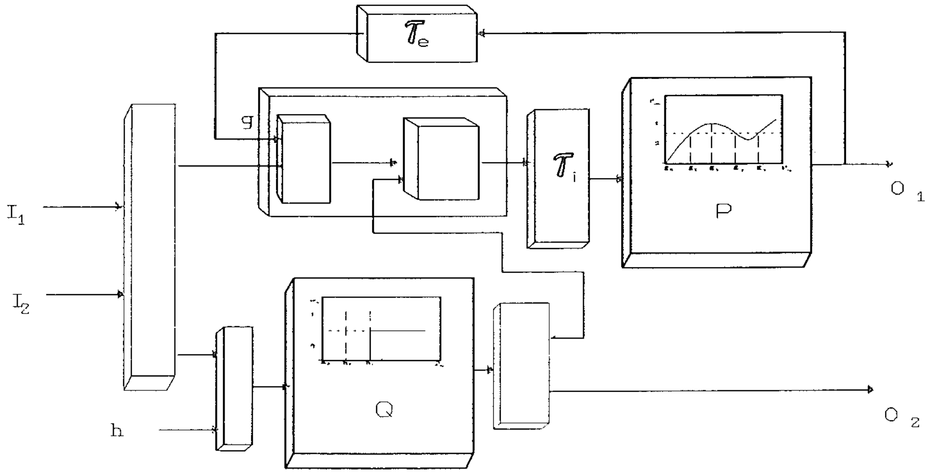

Figure 1.- **Optical-Processing Element with Feedback. White boxes are 2 x 2 or 2 x 1 couplers.**

As it can be shown, there are several possibilities to add feedback to the cell. Any connection between one of the two possible outputs, $\mathbf{O}_1$ or $\mathbf{O}_2$, and any of the four different inputs, namely, $\mathbf{I}_1$, $\mathbf{I}_2$, g and h, should give feedback. But results, depending on the adopted configuration, have to be different. Because the **P**-device output has more possible different output functions, depending on its control signal, namely seven, than the **Q**, its output has been used for feedback. This signal will be the control signal g for device **P**. Figure 1 shows the employed circuit. A computer simulation has been employed for the rest of present work.

3. General behaviour of the cell.

The first analysis that we have performed considered null delay times. This situation has no analytic solution and no data were obtained. The circumstances are strongly different if we introduce finite delay times, namely, internal and external delays.

According to previous studies [4], the situation with more probability to give a periodic or even chaotic solution is when the internal delay time is shorter than the external one. In any case, input has been a regular train of pulses. The input to the non-linear device is a multilevel signal corresponding to the addition of the two periodic inputs. The period of this signal corresponds, in the case studied, to a time of 14 milliseconds.

If the ratio between internal delay time and external delay time is smaller than 1 ms, we obtain a periodic situation. The period of this signal is strongly dependent on the ratio value. In the particular case, where external delay time is 200 ms and internal delay are 2, 4 and 12 ms, obtained results are summarized in Table I. An interesting result is the duplication in period time when the ratio between delays gets smaller. In our case, it goes from 70 to 280. Hence, frequency doubling has been obtained. This result is one of the best indications of a possible route to chaos.

TABLE I.- Characteristics of the output signals, according to the delay times.

t_p	τ_e	τ_i	τ_i/τ_e	Period
14	200	2	0.01	280
14	200	4	0.02	140
14	200	12	0.06	70

Values given at Table I do not correspond to the real transition points between different periods. These values are in a range where the period remains constant. If we calculate the equivalent to the Feigenbaum ratio for the indicated values, a value of 4 is obtained. But if higher order transition points are taken into account, a number, closer to 4.6, has been obtained.

As it is can be seen in Table I, as the internal delay time goes to smaller values, the period of the output signal gets higher and, eventually, becomes chaotic. This situation has been obtained only by computer simulation with internal delay time zero. Experimentally, we have not tried to obtain it. A sample of such a situation is shown in Fig. 2.

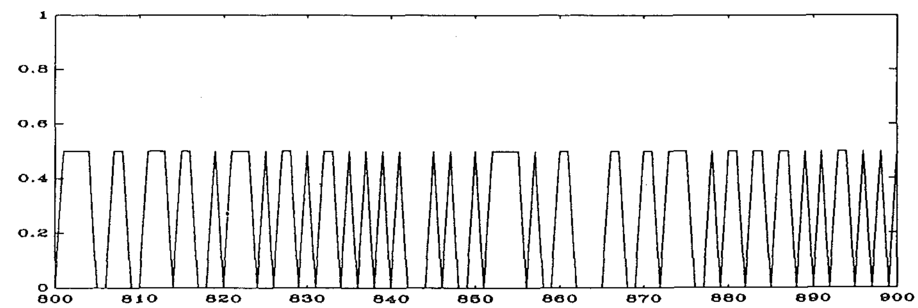

Figure 2.- Output from the logic cell when a chaotic behavior is present.

In order to characterize the obtained chaotic signal, conventional methods are very difficult to apply. A problem related with the above results is the presence of a digital signal with just to two values, "0" and "1". Methods employed with analogue signals are not applicable here. Hence a new technique has to be implemented. The method we have adopted is to group sets of four bits and to convert them to their corresponding hexadecimal value. Hence, for example, "0010" would be a "2", "1001" a "9" and "1110" a "14".

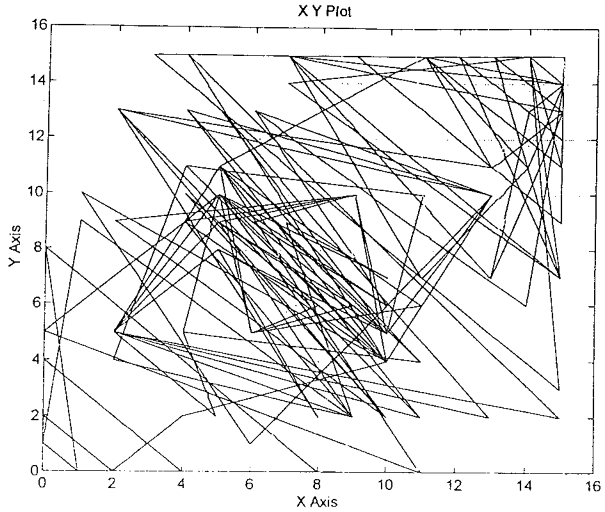

Figure 3.- **Diagram t_{i+1} vs. t_i for a digital chaotic signal as in Fig. 2.**

A diagram, similar to the t_{i+1} versus t_i in analogue signals, has been obtained here. In the case of periodic signals, a regular configuration is obtained. But in the case of chaotic signals, no definite pattern is obtained. This situation appears in Fig 3.

4. Conclusions

A new type of digital chaotic signal has been presented. It is the result of a feedback in an optical processing logic cell, previously reported. According to the reported results, a new possibility to study digital chaos has been employed . It is based in the conversion from binary to hexadecimal signals. Diagrams t_{i+1} vs. t_i can be obtained with our method.

References

[1] **González-Marcos, A. & J.A. Martín-Pereda.** "Quasi-chaotic digital behaviour in an optically processing element". **SPIE**, 2038, 67-77. (1993).

[2] **J.A. Martín-Pereda & A. González-Marcos.** "Optical Programmable Processing Element using Optical Fibers". IEEE Lasers and Electro-Optics Society, **LEOS'92**. Boston, 15-20 November, 1992.

[3] **A. González-Marcos**, *PhD Thesis*. Universidad Politécnica de Madrid. 1993.

[4] **A. Neyer and E. Voges,** "Dynamics of Electrooptic Bistable Devices with Delayed Feedback", **IEEE J. Quantum Electron.**, QE-18, 2009-2015. 1982.